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A NOTE ON ESTIMATING THE MEAN OF SYMMETRICAL POPULATIONS

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SUMMARY

Three parameters family of estimators for μ, the population mean of a symmetrical population have been proposed when its variance σ^2 is (i) unknown, and (ii) known. Then, in particular, subclasses of suggested family of estimators are considered and their properties studied to the second order of approximation. It is found that the subclasses of estimators have the same mean squared error (MSE) but smaller bias than those of other estimators considered by various authors.

Keywords : Three-parameters family; subclasses of estimators; bias; mean squared error.

Introduction

In the literature it is well known that the sample mean is an unbiased estimator of μ, the mean of a symmetrical population with variance σ^2. If one is prepared to sacrifice the unbiasedness property, improved estimators can be obtained. One such estimator was proposed by Searles [2] assuming $C^2 = \sigma^2/\mu^2$, the square of coefficient of variation, to be known. When C^2 is not known a simple alternative is to estimate it from sample. C^2 can be estimated in two ways :

(a) When σ^2 is unknown : (i) $\hat{C}^2 = s^2/\bar{y}^2$; (ii) $\hat{C}^2 = \dfrac{s^2}{\bar{y}^2}\left(1 + \dfrac{s^2}{n\bar{y}^2}\right)^{-1}$,

and

(b) When σ^2 is known : (i) $\hat{C}^2 = \sigma^2/\bar{y}^2$; (ii) $\hat{C}^2 = \dfrac{\sigma^2}{\bar{y}^2}\left(1 + \dfrac{\sigma^2}{n\bar{y}^2}\right)^{-1}$

where $\bar{y} = n^{-1} \sum_{i=1}^{n} y_i$, $s^2 = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \bar{y}^2)$ and n being the sample size. This led few authors to formulate many biased estimators of μ. Keeping in view the form of estimators considered by previous authors, three-parameters family of estimators are proposed :

(i) when σ^2 is unknown

$$d_h = \bar{y} \left[1 + \frac{k s^2}{n\bar{y}^2} \left(1 + \frac{g s^2}{n\bar{y}^2} \right)^{-\alpha} \right], \tag{1.1}$$

(ii) when σ^2 is known

$$d_h^* = \bar{y} \left[1 + \frac{k \sigma^2}{n\bar{y}^2} \left(1 + \frac{g \sigma^2}{n\bar{y}^2} \right)^{-\alpha} \right] \tag{1.2}$$

where k, g and α are the characterizing scalars. Estimators d_h and d_h^* reduce to the set of known estimators for suitable choices of k, g and α. These are shown in the Tables 1 and 2.

To terms of order $O(n^{-2})$ the relative biases [RB $(\cdot)$ = Bias $(\cdot)/\mu$] and relative mean squared errors [RM $(\cdot)$ = MSE $(\cdot)/\mu^2$] of the estimators presented in Tables 1 and 2 are given in Tables 3 and 4 respectively.

In this note we have provided properties of the subclasses d_{h_1} and $d_{h_1}^*$ of d_h and d_h^* respectively in symmetrical population (i.e. in the population where $\sqrt{\beta_1} = \mu_3/\sigma^3 = 0$, μ_3 being the third central moment) to terms of order $O(n^{-2})$ in the subsequent sections. It has been shown that the proposed estimator d_{h1} has the same mean squared errors (MSE) as Sahai and Ray [1], Srivastava and Bhatnagar [7], Upadhyaya and Srivastava [10] and Srivastava and Dwivedi [8] but smaller bias than their estimators. Also the estimator d_{h1} has smaller MSE and bias than that of Srivastava [4]. The proposed estimator d_{h1}^* is superior to those considered by Upadhyaya and Srivastava [11], Upadhyaya and Singh [12] and Upadhyaya and Singh [3].

2. Estimator for μ When σ^2 is Unknown

Setting $k = 1$ and $g = 1$ in (1.1) we obtain a class of estimators of μ as

$$d_{h1} = \bar{y} \left[1 + \frac{s^2}{n\bar{y}^2} \left(1 + \frac{s^2}{n\bar{y}^2} \right)^{-\alpha} \right] \tag{2.1}$$

where α is a characterizing scalar to be chosen suitably. It is to be noted that for $\alpha > 2$, the estimator d_{h1} yields new estimators of μ.

TABLE 1—KNOWN ESTIMATORS FOR μ WHEN σ^2 IS UNKNOWN

Name of Author(s)	*Choice of Characterizing scalars*			*Estimators*
	k	g	α	
Srivastava [4, 5]	-1	1	1	$d_1 = \bar{y}\left[1 - \frac{s^2}{n\bar{y}^2}\left(1 + \frac{s^2}{n\bar{y}^2}\right)^{-1}\right]$
Srivastava [4, 5]	-1	—	0	$d_2 = \bar{y}\left(1 - \frac{s^2}{n\bar{y}^2}\right)$
Thompson [9]	k	$-k$	1	$d_3 = \bar{y}\left[1 + \frac{ks^2}{n\bar{y}^2}\left(1 - \frac{ks^2}{n\bar{y}^2}\right)^{-1}\right]$
Upadhyaya and Srivastava [10]	1	—	0	$d_4 = \bar{y}\left(1 + \frac{s^2}{n\bar{y}^2}\right)$
Sahai and Ray [1]	1	1	1	$d_5 = \bar{y}\left[1 + \frac{s^2}{n\bar{y}^2}\left(1 + \frac{s^2}{n\bar{y}^2}\right)^{-1}\right]$
Srivastava and Banarasi [6]	1	1	2	$d_6 = \bar{y}\left[1 + \frac{s^2}{n\bar{y}^2}\left(1 + \frac{s^2}{n\bar{y}^2}\right)^{-2}\right]$
Srivastava and Bhatnagar [7]	k	g	-1	$d_7 = \bar{y}\left[1 + \frac{ks^2}{n\bar{y}^2}\left(1 + \frac{gs^2}{n\bar{y}^2}\right)^{-1}\right]$
Srivastava and Dwivedi [8]	1	-1	1	$d_8 = \bar{y}\left[1 + \frac{s^2}{n\bar{y}^2}\left(1 - \frac{s^2}{n\bar{y}^2}\right)^{-1}\right]$

TABLE 2—KNOWN ESTIMATORS FOR μ WHEN σ^2 IS KNOWN

Name of Author(s)	*Choice of characterizing scalars*			*Estimators*
	k	g	α	
Upadhyaya and Srivastava [11]	1	—	0	$d_1^* = \bar{y}\left(1 + \frac{\sigma^2}{n\bar{y}^2}\right)$
Upadhyaya and Singh [12]	1	1	1	$d_2^* = \bar{y}\left[1 + \frac{\sigma^2}{n\bar{y}^2}\left(1 + \frac{\sigma^2}{n\bar{y}^2}\right)^{-1}\right]$
Singh and Upadhyaya [3]	1	1	2	$d_3^* = \bar{y}\left[1 + \frac{\sigma^2}{n\bar{y}^2}\left(1 + \frac{\sigma^2}{n\bar{y}^2}\right)^{-2}\right]$
Srivastava and Bhatnagar [7]	k	g	1	$d_4^* = \bar{y}\left[1 + \frac{k\sigma^2}{n\bar{y}^2}\left(1 + \frac{g\sigma^2}{n\bar{y}^2}\right)^{-1}\right]$

TABLE 3—RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_i, $i = 1$ TO 8 IN SYMMETRICAL POPULATION

Estimators	*Relative biases*	*Relative mean squared errors*
d_1	$\mathrm{RB}\,(d_1) = n^{-1} C^2$	$\mathrm{RM}\,(d_1) = n^{-1}C^2\,(1 + 3n^{-1}C^2)$
d_2	$\mathrm{RB}\,(d_2) = -n^{-1}C^2\,(1 + n^{-1}C^2)$	$\mathrm{RM}\,(d_2) = n^{-1}C^2\,(1 + 3n^{-1}C^2)$
d_3	$\mathrm{RB}\,(d_3) = n^{-1}k\,C^2\,[1 + n^{-1}\,(1 + k)C^2]$	$\mathrm{RM}\,(d_3) = n^{-1}C^2\,[1 + n^{-1}k\,(k - 2)C^2]$
d_4	$\mathrm{RB}\,(d_4) = n^{-1}C^2\,(1 + n^{-1}C^2)$	$\mathrm{RM}\,(d_4) = n^{-1}C^2\,[1 - n^{-1}C^2)$
d_5	$\mathrm{RB}\,(d_5) = n^{-1}C^2$	$\mathrm{RM}\,(d_5) = n^{-1}C^2\,(1 - n^{-1}C^2)$
d_6	$\mathrm{RB}\,(d_6) = n^{-1}C^2\,(1 - n^{-1}C^2)$	$\mathrm{RM}\,(d_6) = n^{-1}C^2\,(1 - n^{-1}C^2)$
d_7	$\mathrm{RB}\,(d_7) = n^{-1}k\,C^2\,[1 + n^{-1}\,(1 - g)C^2]$	$\mathrm{RM}\,(d_7) = n^{-1}C^2\,[1 + n^{-1}k\,(k - 2)C^2]$
d_8	$\mathrm{RB}\,(d_8) = n^{-1}C^2\,(1 + 2\,n^{-1}C^2)$	$\mathrm{RM}\,(d_8) = n^{-1}C^2\,(1 - n^{-1}C^2)$

TABLE 4—RELATIVE BIASES AND RELATIVE MEAN SQUARED ERRORS OF d_i^*, $i = 1$ TO 4 IN SYMMETRICAL POPULATION

Estimators	*Relative biases*	*Relative mean squared errors*
d_1^*	$\text{RB}(d_1^*) = n^{-1}C2\,(1 + n^{-1}C2)$	$\text{RM}(d_1^*) = n^{-1}C2\,(1 - n^{-1}C2)$
d_2^*	$\text{RB}(d_2^*) = n^{-1}C2$	$\text{RM}(d_2^*) = n^{-1}C2\,(1 - n^{-1}C2)$
d_3^*	$\text{RB}(d_3^*) = n^{-1}C2\,(1 - n^{-1}C2)$	$\text{RM}(d_3^*) = n^{-1}C2\,(1 - n^{-1}C2)$
d_4^*	$\text{RB}(d_4^*) = n^{-1}k\,C2\,[1 + n^{-1}(1 - g)C2]$	$\text{RM}(d_4^*) = n^{-1}C2\,[1 + n^{-1}k\,(k - 2)C2]$

In order to evaluate the bias and mean squared error of d_{h1}; to terms of order $O(n^{-2})$, let

$$\bar{y} = \mu + U \quad \text{and} \quad s^2 = \sigma^2 + V$$

where U and V are of order $O(n^{-1/2})$ with $E(U) = E(V) = 0$ and also assume that $| U/\mu | < 1$. Then upto the order $O(n^{-2})$, we have

$$d_{h1} = (\mu + U)\left[1 + \frac{(\sigma^2 + V)}{n(\mu + U)^2}\left\{1 + \frac{(\sigma^2 + V)}{n(\mu + U)^2}\right\}^{-\alpha}\right]$$

$$= \mu\left(1 + \frac{U}{\mu}\right)\left[1 + \frac{C^2}{n}\left(1 + \frac{V}{\sigma^2}\right)\left(1 + \frac{U}{\mu}\right)^{-2}\right.$$

$$\left.\times\left\{1 + \frac{C^2}{n}\left(1 + \frac{V}{\sigma^2}\right)\left(1 + \frac{U}{\mu}\right)^{-2}\right\}^{-\alpha}\right]$$

$$= \mu\left[1 + \frac{U}{\mu} + \frac{C^2}{n}\left(1 + \frac{V}{\sigma^2}\right)\left(1 + \frac{U}{\mu}\right)^{-1}\right.$$

$$\left. - \frac{\alpha\, C^4}{n^2}\left(1 + \frac{V}{\sigma^2}\right)^2\left(1 + \frac{U}{\mu}\right)^{-3} + \ldots\right]$$

$$= \mu\left[1 + \frac{U}{\mu} + \frac{C^2}{n}\left\{1 - \frac{U}{\mu} + \frac{V}{\sigma^2} + \frac{U^2}{\mu^2}\right.\right.$$

$$\left.\left. - \frac{UV}{\mu\sigma^2} + \ldots\right\} - \frac{\alpha}{n^2}\, C^4\right]$$

So that

$$E(d_{h1}) = \mu\left[1 + \frac{C^2}{n} + \frac{C^4}{n^2} - \frac{\alpha\, C^4}{n^2}\right]$$

$$\Rightarrow \text{Bias } (d_{h1}) = E(d_{h1} - \mu) = \frac{\mu C^2}{n}\left[1 - (\alpha - 1)\frac{C^2}{n}\right]$$

$$\Rightarrow \text{RB } (d_{h1}) = \frac{\text{Bias } (d_{h1})}{\mu} = \frac{C^2}{n}\left[1 - (\alpha - 1)\frac{C^2}{n}\right], \tag{2.2}$$

for symmetrical populations. In the similar manner, to terms of order $O(n^{-2})$ the mean squared error of d_{h1} is obtained as

$$\text{MSE } (d_{h1}) = \frac{\mu^2 C^2}{n}\left(1 - \frac{C^2}{n}\right) \tag{2.3}$$

$$\Rightarrow \text{RM } (d_{h1}) = \frac{\text{MSE } (d_{h1})}{\mu^2} = \frac{C^2}{n}\left(1 - \frac{C^2}{n}\right) \tag{2.4}$$

From Table 3 and equations (2.2) and (2.4) it is observed that

$$\left.\begin{array}{l} \text{RB}\,(d_{h1}) < \text{RB}\,(d_6) \quad \text{for} \quad \alpha > 2 \\ \text{and} \\ \text{RM}\,(d_{h1}) = \text{RM}\,(d_6) \end{array}\right\} \tag{2.5}$$

From Table 3 it is seen that the estimator d_6 is superior to the estimators d_1, d_2, d_4, d_5 and d_8. Hence, from (2 5) it follows that the proposed estimator d_{h_1} is superior to the estimators d_1, d_2, d_4, d_5, d_6 and d_8 reported by Srivastava [4], Upadhyaya and Srivastava [10], Sahai and Ray [1], Srivastava and Banarasi [6], and Srivastava and Dwivedi [8] respectively.

The percentage reduction in the bias by the proposed estimator d_{h_1}, over d_6 is given by

$$\frac{\text{Bias}\,(d_6) - \text{Bias}\,(d_{h1})}{\text{Bias}\,(d_6)} \times 100$$

$$= \frac{(\alpha - 2)C^2}{(n - C^2)} \times 100 \tag{2.6}$$

From (2.6), it follows that the reduction in bias is large when C^2 is large, meaning thereby that estimator d_{h1} is superior to the estimator d_6, provided $\alpha > 2$ and $n > C^2$. However for smaller sample the reduction in bias is still high.

3 Estimator for μ When σ^2 is Known

Substituting $k = 1$ and $g = 1$ in (1.2) we get a class of estimators for μ as

$$d_{h1}^* = \bar{y}\left[1 + \frac{\sigma^2}{n\bar{y}^2}\left(1 + \frac{\sigma^2}{n\bar{y}^2}\right)^{-\alpha}\right] \tag{3.1}$$

where α is a characterizing scalar to be chosen suitably. It is to bo pointed out that for $\alpha > 2$, the estimator $d_{h_1}^*$ gives new estimators and the estimators proposed by Upadhyaya and singh [12] and Singh and Upadhyaya [3] are the special cases of (3.1).

Proceeding in the similar manner as in section 2, we obtain the bias and mean squared error of d_{h1}^*, to terms of order $O(n^{-2})$, are respectively, given by

$$\text{Bias}\;(d_{h1}^*) = \frac{\mu C^2}{n}\left[1 - (\alpha - 1)\,\frac{C^2}{n}\right]$$

$$\Rightarrow \text{RB}\;(d_{h1}^*) = \frac{C^2}{n}\left[1 - (\alpha - 1)\,\frac{C^2}{n}\right] \tag{3.2}$$

and

$$\text{MSE}\,(d^*_{h1}) = \frac{\mu^2 C^2}{n}\left(1 - \frac{C^2}{n}\right)$$

$$\Rightarrow \text{RM}\,(d^*_{h1}) = \frac{C^2}{n}\left(1 - \frac{C^2}{n}\right) \tag{3.3}$$

From Table 4, equations (3.2) and (3.3) we have

$$\left.\begin{aligned} &\text{RB}\,(d^*_{h1}) < \text{RB}\,(d^*_3) \quad \text{for} \quad \alpha > 2 \\ \text{and} \\ &\text{RM}\,(d^*_{h1}) = \text{RM}\,(d^*_3) \end{aligned}\right\} \tag{3.4}$$

It follows that the proposed estimator d^*_{h1} is superior to the estimator d^*_3 forwarded by Singh and Upadhyaya [3] in the sense that it has lesser bias than that of d^*_3. It is to be noted that the estimator d^*_3 is superior to the estimators d^*_1 and d^*_2 (see Table 4). Hence the proposed estimator d^*_{h1} is also better than those considered by Upadhyaya and Srivastava [11] and Upadhyaya and Singh [12].

The percentage reduction in the bias by the proposed estimator d^*_{h1} over d^*_3 is given by

$$\frac{\text{Bias}\,(d^*_3) - \text{Bias}\,(d^*_{h1})}{\text{Bias}\,(d^*_3)} \times 100$$

$$= \frac{(\alpha - 2)\,C^2}{(n - C^2)} \times 100 \tag{3.5}$$

It follows that the reduction in bias is large when C^2 is large, meaning thereby that the proposed estimator d^*_{h1} is superior to the estimator d^*_3, provided $\alpha > 2$ and $n > C^2$. However for small sample size n the estimator d^*_{h1} considered here is still superior to d^*_3.

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